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Heat exchange between two coupled moving beds by fluid flow

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Abstract

Heat exchange between a moving bed and fluid flow is considered. The arrangement of flows of fluid and solids can be in parallel flow, counter-flow or cross-flow. The effect of possible phase change is considered using a simplified model. Heat exchange by forced fluid flow between two coupled moving beds containing solids can be used to recover heat from hot products to cool input in many industrial processes. There are several ways to couple such two moving beds. Analytical solutions for the fluid and solid temperature distributions and heat recovery effectiveness are presented for continuous operation for single moving beds and two beds coupled by a fluid flow. The intra-particle transient temperature distributions are accounted for in the more accurate analysis and the results are compared to lumped analysis and an approximate solution. The heat recovery effectiveness of a coupled system reaches maximum at certain optimum flow rate of the fluid. Analogy between regenerators and heat recovery systems consisting of two moving beds coupled by an unmixed flow of fluid is found. Due to this analogy many different calculation methods, tables and formulas developed for to evaluate temperatures and thermal effectiveness of regenerators can be used to study heat recovery from solids.

1. Introduction

In many industrial processes solid material is at first heated and possibly melted after which the solids are again cooled to the temperature of the surroundings. There are such heat treatment processes in metal and steel industry. The cooking and boiling of food and the destruction of micro-organisms are examples in food industry. The clay (bricks and ceramics) and glass industries are also examples having such processes. Heat can be recovered from a moving bed to fluid flowing in counter-flow, parallel flow or cross-flow direction against the solids. The heat or mass transfer phenomena in moving beds have been studied to some extent [1–8]. A fluidised bed can also be used to recover heat from hot solid particles [9].

In some cases, the most natural need for the heat recovered would be to preheat the input flow of solids going into the heat treatment of the same process. The energy required in heating of the solids from the ambient temperature to the process temperature is again liberated from the solid material, when it is cooled. The energy demand of the process can be reduced, if part of the heat liberated during cooling can be utilised for preheating of the cool input. The principle is illustrated in Fig. 1. Indeed, theoretically if the system is ideally insulated from the surroundings, the heating power P required approaches zero, when the effectiveness ε approaches unity ($P \to 0$, when $\varepsilon \to 1$) in the case of equal heat capacity flow rates ($\dot{C}_{p1} = \dot{C}_{p2}$). The actual maximum effectiveness of the system is determined in addition to heat losses by the economics of the total system. In such analysis, in addition to the energy saving, the reduction of the size of the actual process heating system should be considered. For example, the elements of the solids in the heat exchanger (Fig. 1) could be hot products of metal or ceramic industry, hot coke, food in packed in tins or glass in a heating process to destroy microbes by heating during sterilisation, etc.

Two types of heat recovery systems are shown in Fig. 2. In the case *a* the heat recovery system is separated

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Nomenclature

a	thermal diffusivity of solid, $a = \lambda_p/(\rho_p c_p)$ [m^2 s^{-1}]
Bi	Biot number, $Bi = hR/\lambda$
C	heat capacity, $C = cm$ [J K^{-1}]
$\dot{C}$	heat capacity flow rate, $\dot{C} = c\dot{m}$ [W K^{-1}]
c	specific heat capacity [J kg^{-1} K^{-1}]
Fo	Fourier number, $Fo = at/R^2$
G''	conductance/surface area [W m^{-2} K^{-1}]
h	heat transfer coefficient [W m^{-2} K^{-1}]
L	length of the moving bed [m]
l_m	specific enthalpy of phase change [J kg^{-1}]
m	mass [kg]
$\dot{m}$	mass flow rate [kg s^{-1}]
P	heating power [W]
R	characteristic length of an element, half thickness of a plate, radius of a cylinder or a sphere [m]
R_*	ratio of heat flows, $R_* = l_{m,e}\dot{m}_p/[\dot{C}_f(T_m - T_{f,in})]$
R_f	ratio of heat capacity flow rates, $R_f = \dot{C}_f/\dot{C}_p$
R_m	distance of phase change front from centre of an element [m]
S	heat transfer area [m^2]
s	Laplace transform variable
T	temperature [K]
t	time [s]
$\dot{V}$	volumetric flow rate [m^3 s^{-1}]
w	velocity of solids, $w = L/t_r$ [m s^{-1}]
X	ratio of radius, $X = R_m/R_p$
x	co-ordinate normal to the surface of the element, moving coodinate system [m]
y	co-ordinate along flow of solids in parallel- and counter-flow, $y = wt$ [m]
z	co-ordinate along the flow of solids in cross-flow, $z = wt$ [m]

Greek symbols

Γ	geometry factor, 0 for a plate, 1 for a cylinder, 2 for a sphere
ε	thermal effectiveness of heat recovery
ς	dimensionless co-ordinate normal to the element surface, $\varsigma = x/R$
η	dimensionless time, $\eta = hSt/C_p$
ϑ	dimensionless temperature of fluid, $\vartheta = (T_f - T_{p,in})/(T_{f,in} - T_{p,in})$ for single bed, $\vartheta = (T_f - T_{p1,in})/(T_{p2,in} - T_{p1,in})$ for two coupled beds
ϑ^*	dimensionless temperature of fluid, $\vartheta^* = (T_f - T_m)/(T_m - T_{f,in})$
θ	dimensionless temperature of solid, $\theta = (T_p - T_{p,in})/(T_{f,in} - T_{p,in})$ for single bed, $\theta = (T_p - T_{p1,in})/(T_{p2,in} - T_{p1,in})$ for two coupled beds
Θ	dimensionless temperature of solid, $\Theta = (T_p - T_{p,min})/(T_{p,max} - T_{p,min})$
Λ	dimensionless length, dimensionless heat transfer area, $\Lambda = h_e S/\dot{C}_f$
λ	thermal conductivity [W m^{-1} K^{-1}]
ξ	dimensionless co-ordinate along the fluid flow, $\xi = (hS/\dot{C})y/L$
Π	dimensionless residence time of solids, $\Pi = h_e S t_r/C_p$
ϕ	shape coefficient for internal conduction, $\phi = 1/(2\Gamma + 6)$

Subscripts

0	at inlet of solids
1	bed with cooler inlet solid temperature, first value in series
2	bed with hotter inlet solid temperature
a	average
c	cycle
e	effective, divided by $1 + 2\phi Bi$
f	fluid
in	inlet, inlet of the lower temperature for two beds
m	phase change
max	related to solids with maximum heat capacity flow rate
min	related to solids with minimum heat capacity flow rate
out	outlet
p	solid
r	residence
s	surface
t	total

from the heating process. Different fluid is applied in the heat recovery than that used in the heat treatment (boiler flue gases, steam or hot liquid from a heat source). Also direct heating with electric energy can be applied. In the case *b* the combustion air and the flue gas flows of the heat treatment system are also used in the heat recovery zone.

Fig. 3a illustrates a possible solids heat exchanger of the indirect type. Fluid circulation between two moving beds can be used to recover heat from hot products in a moving bed (or on a moving grate) to heat the cool input in another moving bed in a industrial continuous heating process. There is optimum flow rate of fluid in order to maximise the heat recovery efficiency in counter-flow,

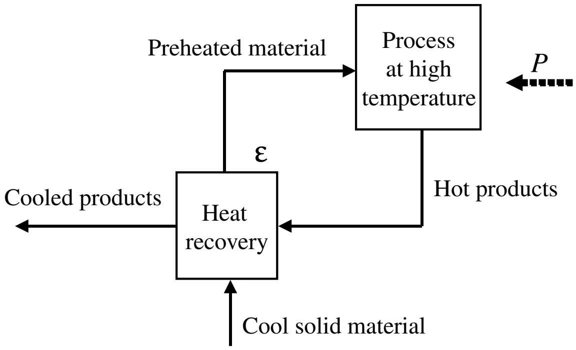

Fig. 1. Heat recovery between solids in high temperature process.

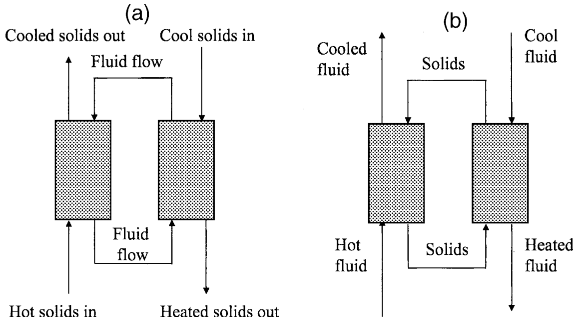

Fig. 3. Two coupled moving beds. (a) Heat exchange between two solid flows by a fluid flow. (b) Heat exchanger for fluids using solids as the heat carrier.

but besides that other things, such as the heating or cooling rate required for the process due to product quality may be important. The flows of solids and fluid are in counter-flow in this illustration, which is most effective. Parallel-flow is less effective, but gives more rapid cooling. Cross-flow is also possible. In principle, flow of solids can be used as a heat carrier and as a cheap heat transfer surface area to exchange heat between two flows of fluid as shown in Fig. 3b. For example sand could be a cheap and attractive heat carrier [10].

Two examples of two coupled beds with solids and fluid in cross-flow and with fluid unmixed between the moving beds, which is more effective for heat recovery, are shown in Fig. 4. The case *a* is much analogous to a recuperative heat exchanger for fluids. At high temperatures it is also possible to use direct radiation between two flows of solids without fluid flow to recover heat from hot products to heat up cold input or to produce steam [11]. In the case *b* the heat exchange between the two beds is described by analogous equations to regenerative heat exchangers as will be shown.

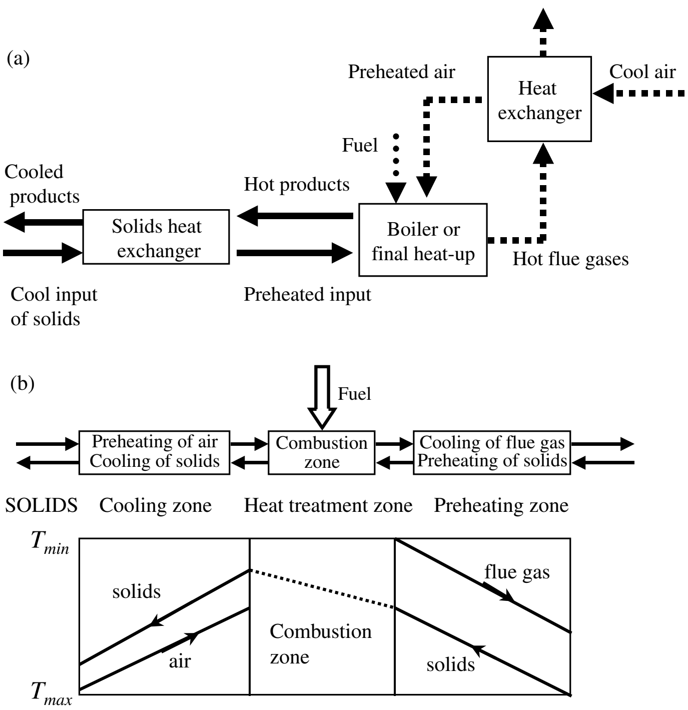

Fig. 2. Indirect (a) and direct (b) heat recovery systems.

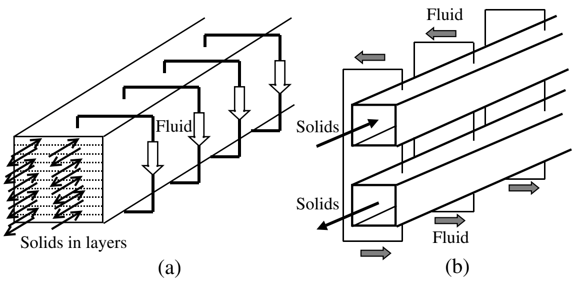

Fig. 4. Two types of heat recovery between solids in counter-flow (fluid unmixed in the direction of the solids). (a) A system analogous to a recuperative heat exchanger. (b) A system analogous to a regenerative heat exchanger.

There is a wide literature on heat exchangers for fluids. Much less studies concern with heat exchange between fluid and solids. Many equations developed for heat exchange between fluids can be modified to apply for heat exchange between fluid and solids. In an earlier paper [12], heat exchange between two fixed beds coupled with a fluid flow was studied. This paper deals with two moving beds coupled with a fluid flow.

2. Single moving bed

At first a single moving bed of solids is considered. Usually it is assumed that the thermal resistance of the solids can be neglected or that the conduction is lumped into effective thermal conductance, but also intra-particle conduction effect has been considered [1,3] for spherical solids. Here also other one-dimensional cases are studied analytically and the treatment can be extended to other regular shapes (cubes, parallelepiped solids, finite cylinders). The heat conduction and storage inside the single solid bed elements is described by the Fourier equation and convection boundary condition

$$\frac{\partial T_{\mathrm{p}}}{\partial t} = a_{\mathrm{p}}\nabla^2 T_{\mathrm{p}}, \quad h(T_{\mathrm{f}} - T_{\mathrm{s}}) = \lambda_{\mathrm{p}}\left(\frac{\partial T_{\mathrm{p}}}{\partial x}\right)_{\mathrm{s}} \tag{1}$$

The average heat transfer coefficient around the bed element surface is applied. The moving beds are assumed to be ideally insulated and the flow across the bed distributed evenly. Then there are no temperature differences normal to the flow direction in parallel- and counter-flow. Amundson [1] studied the non-insulated moving bed with temperature distribution across the flow.

2.1. Parallel flow and counter-flow of fluid and solids

The effect of axial conduction or mixing is considered insignificant compared to the convection heat flow in the analysis. Constant solid and fluid properties as well as uniform velocities are assumed. The element characteristic size that determines the heating rate of the element is assumed to be small compared to the length of the moving bed and it can be considered differential. Then the heat transfer in the fluid is described by

$$\mp \dot{C}_{\mathrm{f}}\frac{\partial T_{\mathrm{f}}}{\partial y} - (C_{\mathrm{f}}/L)\frac{\partial T_{\mathrm{f}}}{\partial t} = h(S/L)(T_{\mathrm{f}} - T_{\mathrm{s}}) = (C/L)\frac{\mathrm{d}T_{\mathrm{p,a}}}{\mathrm{d}t} \tag{2}$$

where the upper sign is for parallel flow and lower for counter-flow (also in later equations). The first term on the left-hand side accounts for the heat carried with the fluid flow and the second is due to heat storage in the fluid. The time t from the instant for solid entering to the moving bed is related to the space co-ordinate by $y = wt$. In steady state, the local fluid temperature is constant, and the second term is zero. The right-hand side equals the convection heat exchange between the fluid and the surface of the solids, which is also equal to the energy storage in the solids described by the last term. The heat transported by conduction and mixing in the flow direction is assumed to be insignificant compared to the forced convection flow. The inlet condition for the solid is $T_{\mathrm{p}}(y=0) = T_{\mathrm{p,in}}$. The inlet condition for the fluid is $T_{\mathrm{f}}(y=0) = T_{\mathrm{f,in}}$ for parallel flow and $T_{\mathrm{f}}(y=L) = T_{\mathrm{f,in}}$ for counter-flow. Replacing y in Eq. (2) by wt, Eqs. (1) and (2) can be presented in dimensionless form for one-dimensional cases and steady state

$$\frac{\partial\theta}{\partial Fo} = \frac{1}{\varsigma^{\Gamma}}\frac{\partial}{\partial\varsigma}\left(\varsigma^{\Gamma}\frac{\partial\theta}{\partial\varsigma}\right), \quad \vartheta - \theta_{\mathrm{s}} = \frac{1}{Bi}\left(\frac{\partial\theta}{\partial\varsigma}\right)_{\mathrm{s}} = \frac{\mp R_{\mathrm{f}}}{Bi(1+\Gamma)}\frac{\partial\vartheta}{\partial Fo} \tag{3}$$

Applying the Laplace transform

$$\bar{f}(s) = \int_0^{\infty} \mathrm{e}^{-sFo} f(Fo)\,\mathrm{d}Fo \tag{4}$$

Eqs. (3) are transformed into

$$s\bar{\theta} = \frac{1}{\varsigma^{\Gamma}}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\varsigma^{\Gamma}}\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}\varsigma}\right),$$
$$\bar{\vartheta} - \bar{\theta}_{\mathrm{s}} = \frac{1}{Bi}\left(\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}\varsigma}\right)_{\mathrm{s}} = \frac{\mp R_{\mathrm{f}}}{Bi(1+\Gamma)}(s\bar{\vartheta} - \vartheta_0) \tag{5}$$

where ϑ_0 is the dimensionless temperature of the fluid at the inlet of solids. The solution of the transformed solid temperature can be presented in the form [12] $\bar{\theta} = \bar{\theta}_{\mathrm{s}}\bar{m}(\varsigma, s)$, where $\bar{m}(\varsigma, s) = \varsigma^{-(\Gamma-1)/2}J_{(\Gamma-1)/2}(\mathrm{i}\varsigma\sqrt{s})/J_{(\Gamma-1)/2} \times (\mathrm{i}\sqrt{s})$ and J is Bessel function of the first kind. The transformed element surface temperature can be related to the transformed fluid temperature [12] $\bar{\theta}_{\mathrm{s}} = [1 - \bar{h}(s)]\bar{\vartheta}$, where $\bar{h}(s) = \bar{m}'(1,s)/[Bi + \bar{m}'(1,s)]$ and $\bar{m}'(1,s) = -\mathrm{i}\sqrt{s}J_{(\Gamma+1)/2}(\mathrm{i}\sqrt{s})/J_{(\Gamma-1)/2}(\mathrm{i}\sqrt{s})$. The solution of the differential equation for the fluid is readily found. The transformed fluid temperature is

$$\bar{\vartheta} = \vartheta_0 R_f / \{R_f s \pm Bi(1+\Gamma)\bar{h}(s)\} \tag{6}$$

The inverse can be found by applying the calculus of residues. The series expression $\bar{h}(s) = s_* - a_2 s_*^2 + a_3 s_*^3 - \cdots$, where $s_* = s/[Bi(1+\Gamma)]$, $a_2 = 1 + Bi/(3+\Gamma)$ and $a_3 = 1 + 2Bi/(3+\Gamma) + 2Bi^2/[(3+\Gamma)(5+\Gamma)]$ is useful in the derivation of the solution for the case $R_f = 1$ for counter-flow. The result is for fluid temperature

$$\vartheta = \vartheta_0 f(Fo) \tag{7}$$

where

$$f(Fo) = C_0 + C_1 Fo + \sum_{n=1}^{\infty} \frac{R_f \exp(s_n Fo)}{R_f \pm Bi(1+\Gamma)\bar{h}'(s_n)} \tag{8}$$

For parallel flow $\vartheta_0 = 1$, $C_0 = R_f/(R_f+1)$ and $C_1 = 0$. For counter-flow $\vartheta_0 = 1/f(Fo_r)$, $C_0 = a_3/a_2^2$ and $C_1 = (1+\Gamma)Bi/a_2$, when $R_f = 1$. $C_0 = R_f/(R_f - 1)$ and $C_1 = 0$, when $R_f \neq 1$. s_n are the real roots of the transcendental equation

$$R_f s_n \pm Bi(1+\Gamma)\bar{h}(s_n) = 0 \tag{9}$$

The corresponding solutions for the temperature of the solids can readily be found. The thermal effectiveness of the heat recovery of a single moving bed is defined as the temperature change in the fluid temperature in the moving bed divided by the difference in the inlet temperatures, $\varepsilon_f = |(T_{f,out} - T_{f,in})/(T_{p,in} - T_{f,in})|$. The thermal effectiveness becomes

$$\varepsilon_f = 1 - [f(Fo_r)]^{\pm 1} \tag{10}$$

The corresponding thermal effectiveness for solids (temperature difference of solids in the bed divided by the difference in the inlet temperatures) is $\varepsilon_s = \varepsilon_f R_f$.

2.2. Cross-flow

For cross-flow the solids are flowing in the direction of the co-ordinate $z = wt$ and it is assumed that the conditions are such that the fluid is approximately flowing in perpendicular direction along co-ordinate y. The energy equation for the fluid is

$$-\dot{C}_f \frac{\partial T_f}{\partial y} - (C_f/L)\frac{\partial T_f}{\partial t} = h(S/L)(T_f - T_s) \tag{11}$$

In steady state, the second term on the left-hand side is zero. Then the energy equation for the fluid and its transformed form in dimensionless form become

$$-\frac{\partial \vartheta}{\partial \xi} = \vartheta - \theta_s, \quad -\frac{d\bar{\vartheta}}{\partial \xi} = \bar{\vartheta} - \bar{\theta}_s = \bar{h}(s)\bar{\vartheta} \tag{12}$$

respectively. The dimensionless inlet conditions are $\vartheta(\xi = 0, Fo) = 1$ and $\theta(\xi, Fo = 0) = 0$. There is an analogy in heat transfer between steady state cross-flow moving bed and transient heating of a fixed bed. Then the solutions for the fixed bed [12] can be applied (with simplification $\gamma = C_f/C_p = 0$ due to steady state in cross-flow). The solution of the transformed fluid temperature becomes

$$\bar{\vartheta} = \exp[-\bar{h}(s)\xi]/s \tag{13}$$

The fluid temperature distribution is obtained as inverse transform. An exact solution in integral form has been presented earlier for this transform [12]. The following approximate solutions [12] are more easily applicable for calculations

$$\vartheta = G_0(\xi_*, Fo_*)U(Fo_*) \tag{14}$$

$$\theta_a = \{K_1[1 - G_0(Fo_*, \xi_*)] + (1 - K_1)G_0(\xi_*, Fo_*)\}U(Fo_*) \tag{15}$$

where $\xi_* = K_1\mu_1^2\xi/[Bi(1+\Gamma)]$ and $Fo_* = \mu_1^2\{Fo - (1 - K_1)\xi/[Bi(1+\Gamma)]\}$. $U(Fo_*)$ is Heaviside's unit step function, $U(Fo_*) = 1$, when $Fo_* \geqslant 0$ and $U(Fo_*) = 0$ when $Fo_* < 0$. μ_1 is the first root of the transcendental equation $\mu_n J_{(\Gamma+1)/2}(\mu_n) = Bi J_{(\Gamma-1)/2}(\mu_n)$ and the coefficient is $K_1 = 2(1+\Gamma)Bi^2/\{\mu_1^2[\mu_1^2 + Bi(1 - \Gamma + Bi)]\}$. The function $G_k(x, y)$ is

$$G_k(x,y) = e^{-x}\sum_{n=0}^{\infty} g_n(x)\frac{y^{n+k}}{(n+k)!} \tag{16}$$

Note earlier misprint in the summation (Eq. (A.3) [12]), which should start from $n = 0$. $g_0(x) = 1$, $g_1(x) = x$ and $g_{n+1}(x) = [(x - 2n)g_n(x) + (1 - n)g_{n-1}(x)]/(1 + n)$, when $n > 1$, are functions related to Laquerre polynomials. Other forms for $G_k(x, y)$ have been reported [13].

The effectiveness of the heat exchange of the system can be defined as the heat transferred from the fluid to the solids divided by the maximum possible heat to be transferred, when $\xi_* = \xi_*(y = L)$,

$$\begin{aligned}\varepsilon_f &= 1 - \frac{1}{Fo_t}\int_0^{Fo_t} \vartheta \, dFo \\ &= 1 - \frac{1}{\mu_1^2 Fo_t} G_1(\xi_*, Fo_*)U(Fo_*)\end{aligned} \tag{17}$$

2.3. Lumped analysis for parallel- and counter-flow

In lumped analysis the resistance due to heat conduction inside the solid is accounted for using an effective heat transfer coefficient $h_e = h/(1 + 2\phi Bi)$ [14]. Then the equations describing the heat exchange between solids and fluid are reduced to the lumped equations

$$\mp\dot{C}_f \frac{dT_f}{dy} = h_e(S/L)(T_f - T_p) = \dot{C}_p \frac{dT_p}{dy} \tag{18}$$

with $T_p(y = 0) = T_{p,in}$, $T_f(y = 0) = T_{f,in}$ for parallel flow and $T_f(y = L) = T_{f,in}$ for counter-flow. The moving bed is analogous to a recuperative heat exchanger. It is a heat exchange system, where the other flow consists of solid matter. The surface of solids acts as the area for heat transfer. The thermal effectiveness of the parallel- and counter-flow recuperators is well-known,

$$\varepsilon_{\rm f} = 1 - (\{\exp[-\Lambda(1 \pm R_{\rm f})] \pm R_{\rm f}\}/(1 \pm R_{\rm f}))^{\pm 1} \tag{19}$$

which for counter-flow as the limit when $R_{\rm f} \to 1$ becomes $\varepsilon_{\rm f} = \Lambda/(1+\Lambda)$.

2.4. Lumped analysis for cross-flow

For cross-flow the heat transfer is described by

$$-\dot{C}_{\rm f}\frac{\partial T_{\rm f}}{\partial y} = h_{\rm e}(S/L)(T_{\rm f} - T_{\rm p}) = (C_{\rm p}/L)\frac{\partial T_{\rm p}}{\partial t} \tag{20}$$

which is in dimensionless form

$$-\frac{\partial \vartheta}{\partial \xi_{\rm e}} = \vartheta - \theta = \frac{\partial \theta}{\partial \eta_{\rm e}} \tag{21}$$

The solutions for the temperatures of solids and fluid and the effectiveness are analogous to the cross-flow heat exchanger and well-known (see e.g. [13,15,16])

$$\theta = 1 - G(\eta_{\rm e}, \xi_{\rm e}), \quad \vartheta = G(\xi_{\rm e}, \eta_{\rm e}),$$
$$\varepsilon_{\rm f} = 1 - G_1(\Lambda, \Pi)/\Pi \tag{22}$$

If the bed is thin, the temperature of the solids is approximately constant in the direction (y) of the fluid and the solutions become

$$\vartheta = \exp(-\xi_{\rm e}) + [1 - \exp(-\xi_{\rm e})] \times \{1 - \exp[-(1 - \exp(-\Lambda))\eta_{\rm e}/\Lambda]\} \tag{23}$$

$$\theta = 1 - \exp\{-[1 - \exp(-\Lambda)][\eta_{\rm e}/\Lambda]\} \tag{24}$$

$$\varepsilon_{\rm f} = \{1 - \exp[-(1 - \exp(-\Lambda))\Pi/\Lambda]\}\Lambda/\Pi \tag{25}$$

2.5. Moving bed with phase change

In food industry, the food can be frozen or melted in a parallel or counter-flow process. In such freezing process, the sensible heat is much less that the heat of phase change. In the following analysis, the sensible heat is approximately accounted for by using an effective heat of phase change $l_{\rm m,e} \approx l_{\rm m} + c_p(T_{\rm p,in} - T_{\rm m})$, where the effect of a possible container wall is included in specific heat capacity c_p. Then heat transfer can be treated with a shrinking core model. The following theory is a simplification of a more complex analogous physical problem [6], which includes convection flow inside the solids. A somewhat analogous phenomena concerning a moving bed sorption system has been considered [17,18]. The thermal balance for a moving bed with freezing or melting leads to

$$-\dot{C}_{\rm f}\frac{{\rm d}T_{\rm f}}{{\rm d}y} = G''(S/L)(T_{\rm f} - T_{\rm p}) = \pm\rho_{\rm p} l_{\rm m,e}\frac{{\rm d}\dot{V}_{\rm m}}{{\rm d}y} \tag{26}$$

The first term is the heat gained or lost by the fluid. It is equal to heat exchange between the fluid and solids shown by the second term and finally consumed in the phase change denoted by the third term. The upper sign is for parallel flow and other is for counter-flow. The heat transfer inside the element is described as conduction to the shrinking unreacted core. Then the conductance as combined convection and conduction is $G'' = h/[1 + Bi\Phi(X)]$, where $X = R_{\rm m}/R_{\rm p}$. The function describing the thermal resistance between surface and phase change front is for one-dimensional elements $\Phi(X) = \int_X^1 x^{-\Gamma}\,{\rm d}x$, $\Phi(X) = 1 - X$ for plates, $\Phi(X) = -\ln(X)$ for cylinders and $\Phi(X) = 1/X - 1$ for spheres. The volume flow rate of material available to phase change locally is related to the distance of the phase change front from the centre of the particle by $\dot{V}_{\rm m} = \dot{V}_{\rm p}(R_{\rm m}/R_{\rm p})^{1+\Gamma}$. Eq. (26) can be presented in dimensionless from

$$-\frac{{\rm d}\vartheta^*}{{\rm d}\xi} = \frac{\vartheta^*}{1 + Bi\Phi(X)} = \pm R_*\frac{{\rm d}X^{1+\Gamma}}{{\rm d}\xi} \tag{27}$$

The inlet condition is $\vartheta^*(\xi = 0) = -1$. At the inlet $X(\xi = 0) = X_0$, where $X_0 = 1$ for parallel flow and unknown (so far) for counter-flow. The fluid temperature can be solved giving

$$\vartheta^* = \pm R_*(X_0^{1+\Gamma} - X^{1+\Gamma}) - 1 \tag{28}$$

and it can be substituted into Eq. (27), yielding

$$\xi = \pm(1+\Gamma)R_*\int_{X_0}^{X}\frac{X^{\Gamma}[1 + Bi\Phi(X)]}{\pm R_*(X_0^{1+\Gamma} - X^{1+\Gamma}) - 1}\,{\rm d}X \tag{29}$$

The location of complete phase change is obtained by substituting $X = 0$. The value of X_0 for counter-flow is determined from this relation, since at the fluid outlet $(y = L, \xi = hS/\dot{C})X = 1$. Eq. (29) can be integrated for plates

$$\xi = Bi(X - X_0) - (1 + Bi - BiX_0 \pm Bi/R_*) \times \ln[1 \pm R_*(X - X_0)] \tag{30}$$

and spheres

$$\xi = \sqrt{3}Bi\kappa\{\arctan[(1 + 2X\kappa)/\sqrt{3}] - \arctan[(1 + 2X_0\kappa)/\sqrt{3}]\} + 1/2Bi\kappa\ln\left(\frac{1 + X_0\kappa + X_0^2\kappa^2}{1 + X\kappa + X^2\kappa^2}\frac{(1 - X\kappa)^2}{(1 - X_0\kappa)^2}\right) + (Bi - 1)\ln[1 \pm R_*(X_0^3 - X^3)] \tag{31}$$

where $\kappa = 1/(X_0^3 \pm 1/R_*)^{1/3}$, respectively. For cylinders the integral in Eq. (29) must be calculated numerically. The solution for the cross-flow situation is analogous to the fixed bed discussed elsewhere [19].

3. Two moving beds coupled with a fluid flow

3.1. Parallel- and counter-flow

When two such heat exchangers are coupled (Fig. 3a), the inlet and outlet temperatures are related by the equations

$$\vartheta_1(0) = \vartheta_2(Fo_1), \quad \vartheta_2(0) = \vartheta_1(Fo_1),$$
$$\theta_1(0) = 0, \quad \theta_2(0) = 1 \tag{32}$$

The equations are also valid for the other case with solids as the heat carrier (Fig. 3b), if temperatures ϑ and θ are exchanged. The thermal effectiveness of coupled moving bed defined as $\varepsilon_i = |\Delta T_{p,i}|/(T_{p2,in} - T_{p1,in})$ is for bed $i = 1$

$$\varepsilon_1 = \theta_1/\theta_2 = R_{1f}/(1/\varepsilon_{1f} + 1/\varepsilon_{2f} - 1) \tag{33}$$

The solution applies for the general case $Bi_1 \neq Bi_2$, $Fo_1 \neq Fo_2$.

The effectiveness of two coupled moving beds reaches maximum with a specific flow rate of the heat carrier. The optimum ratio R_{1f} is found from the condition $d\varepsilon_1/dR_{1f} = 0$, but it seems impossible to solve this in the exact case analytically. Then the optimum can be found by changing gradually the value of R_{1f} and by evaluating the effectiveness. When lumped parameters are used, in the case $\dot{C}_{p1} = \dot{C}_{p2} = \dot{C}_p$, the optimum is with $\dot{C}_f = \dot{C}_p$ i.e. $R_f = 1$ for counter-flow in analogy with the recuperative heat exchangers [15,21,22]. Further optimisation accounting for the dependence of the overall heat transfer coefficient on mass flow rate has been studied [23]. When $R_p = \dot{C}_{p,min}/\dot{C}_{p,max} < 1$, the optimum heat capacity flow rate of the connecting flow $\dot{C}_f$ in counter-flow should be chosen so that [15,24]

$$\dot{C}_{p,min}/\dot{C}_f = (\Lambda_{min} + R_p\Lambda_{max})/(\Lambda_{min} + \Lambda_{max}) \tag{34}$$

3.2. Cross-flow with fluids mixed

Eq. (33) is also valid for cross-flow if the fluids are completely mixed between the solid flows. Thus, for example in the lumped case, the effectiveness becomes

$$\varepsilon = (\Pi_1/\Lambda_1)/\{1/[1 - G_1(\Lambda_1,\Pi_1)/\Pi_1] + 1/[1 - G_1(\Lambda_2,\Pi_2)/\Pi_2] - 1\} \tag{35}$$

In the balanced case ($\Lambda_1 = \Lambda_2$, $\Pi_1 = \Pi_2$) this solution is simplified into

$$\varepsilon = (\Pi/\Lambda)[1 - G_1(\Lambda,\Pi)/\Pi]/[1 + G_1(\Lambda,\Pi)/\Pi] \tag{36}$$

The maximum effectiveness is obtained from the relationship $d\varepsilon/d\Lambda = 0$ giving

$$1 - [G_1(\Lambda,\Pi)/\Pi]^2 + 2G_1(\Lambda,\Pi)/\Pi - 2G(\Lambda,\Pi) = 0 \tag{37}$$

The solution is approximately $\Pi \cong 2 + \Lambda$, when $\Lambda > 5$. In the case of thin beds, the effectiveness of single bed is obtained from Eq. (25) and for two coupled beds ε_f is defined by Eq. (33). In the symmetric and balanced case

$$\varepsilon = 1/\{2/[1 - e^{-(1-e^{-\Lambda})\Pi/\Lambda}] - \Lambda/\Pi\} \tag{38}$$

A relation for the maximum effectiveness is obtained from $d\varepsilon/dR_f = 0$, where $R_f = \Pi/\Lambda$, giving $1 + [1 - \exp(-\Lambda)]R_f^2 = \cosh\{[1 - \exp(-\Lambda)]R_f\}$. The maximum effectiveness $\varepsilon \approx 0.56$ is obtained at about $R_f = \Pi/\Lambda \approx 3$, when $\Lambda > 5$.

3.3. Cross-flow with fluids unmixed between moving beds

The arrangement of the moving bed and the fluid in cross-flow is shown in Fig. 4b. There is symmetry in Eq. (21) for the fluid and solid temperatures. The situation resembles much the operation of a regenerator. There is also symmetry both in the flows of fluid and solids and in the boundary conditions between the system of two moving beds and a regenerator. For two moving beds in counter-flow (Fig. 4b) the boundary conditions are

$$\theta_1(\xi_{e1}, \eta_{e1} = 0) = 0, \quad \theta_2(\xi_{e2}, \eta_{e2} = 0) = 1,$$
$$\vartheta_1(\xi_{e1} = \Lambda_1, \eta_{e1}) = \vartheta_2(\xi_{e2} = 0, \Pi_2 - \eta_{e2}),$$
$$\vartheta_1(\xi_{e1} = 0, \Pi_1 - \eta_{e1}) = \vartheta_2(\xi_{e2} = \Lambda_2, \eta_{e2}) \tag{39}$$

It is interesting to note that the heat recovery system is described by analogous equations and boundary conditions as a regenerator operating in the counter-flow mode. There is a vast literature on the methods for simulating the operation of the counter-flow regenerator and calculation of the thermal effectiveness [14,25]. Then thermal effectiveness of the cross-flow arrangements of two moving beds can be obtained from tables calculated for regenerators, when the parameters Λ and Π are exchanged. Due to this symmetry, the numerical or analytical methods developed for calculation of temperatures in a regenerator can be applied to analyse temperatures in systems of two moving beds. This is done just by exchanging fluid and solid temperatures, ϑ and θ, and the dimensionless time and space co-ordinates η_e and ξ_e. The symmetry relations make it possible to utilise much of existing literature in the planning of the heat recovery systems. For example in the symmetric and balanced case ($\Lambda = \Lambda_1 = \Lambda_2$, $\Pi = \Pi_1 = \Pi_2$) the approximate solution that has been presented for the regenerator [20] is transformed into

$$\varepsilon = \{1 - \Lambda^2[1 - \exp(-\Pi)]/[6\Pi(2 + \Pi)]\}\Pi/(2 + \Pi) \tag{40}$$

for a system of two moving beds giving good accuracy when $\Lambda \leqslant 2$.

4. Discussion

4.1. Single moving bed

As an example a single moving bed in counter-flow with fluid with plate shaped material ($\Gamma = 0$) is considered. In this case

$$\bar{h}(s) = \sqrt{s}\tanh\sqrt{s}/(Bi + \sqrt{s}\tanh\sqrt{s}) \quad (41)$$

and Eq. (9) becomes

$$\tan(q_n)/[Bi - q_n\tan(q_n)] = (R_f/Bi)q_n \quad (42)$$

where $s_n = -q_n^2$, except, when $n = 1$ and $R_f \geqslant 1$

$$\tanh(q_1)/[Bi + q_1\tanh(q_1)] = (R_f/Bi)q_1 \quad (43)$$

where $s_1 = q_1^2$. In the example we choose $R_f = 0.8$ and $Bi = 0.1$. The first values become $q_1 = 0.1555$, $q_2 = 3.173$, $q_3 = 6.299$, $q_4 = 9.435$, $\bar{h}'(q_1) = 6.400$, $\bar{h}'(q_2) = 32006$. The exact solution in counter-flow becomes $\varepsilon_f = 0.75683$. If *Fo* is large enough, only the first term in the summation in Eq. (8) is needed, since the exponential term becomes low for larger $n\,(q_{n+1} \approx q_n + \pi)$. The approximation as a usual recuperative heat exchanger using effective heat transfer coefficient between flows with $\Lambda = (1+\Gamma)Fo_{r,1}(Bi/R_f)/(1+2\phi Bi) = 2.4194$ gives $\varepsilon_f = 0.75679$.

The dependence of the thermal effectiveness of single moving bed as function of *Fo* is shown in Fig. 5. The error between the exact, Eq. (10), and the lumped model, Eq. (19), is shown in Fig. 6. Another better approximation is derived here. By applying the approximation [12] $Bi(1+\Gamma)\bar{h}(s) \approx K_1\mu_1^2 s/(s+\mu_1^2) + (1-K_1)s$ in Eq. (6) it is possible to derive the simplified approximate solution for $f(Fo)$

$$f(Fo) = \frac{\pm R_f}{1 \pm R_f}\left[1 + \frac{K_1}{1 - K_1 \pm R_f} \times \exp\left(-\frac{(1 \pm R_f)\mu_1^2 Fo}{1 - K_1 \pm R_f}\right)\right] \quad (44)$$

that can be used for evaluate approximate fluid temperature, Eq. (7), or the thermal effectiveness, Eq. (10). For counter-flow, when $R_f = 1$, the simple linear relation $f(Fo) = (1 + \mu_1^2 Fo)/K_1$ is obtained as the limit ($R_f \to 1$) from Eq. (44).

In the preceding example case approximate solution, Eq. (44), gives the same value 0.75683 with 5 first digits agreeing with the exact solution. The approximate solution is more accurate than the lumped one. It can be seen (Fig. 6) that the errors are great with small values of *Fo* and large values of *Bi*. As an example, when $Bi = 10$, $Fo = 0.2$ and $R_f = 0.8$, Eq. (10) with the exact $f(Fo)$ defined by Eq. (8) gives $\varepsilon_f = 0.4415$ (or 0.4426 only with the first term). With approximate $f(Fo)$ defined by Eq. (44), $\varepsilon_f = 0.4607$. The lumped model, Eq. (19), gives 0.3795 and a poor accuracy.

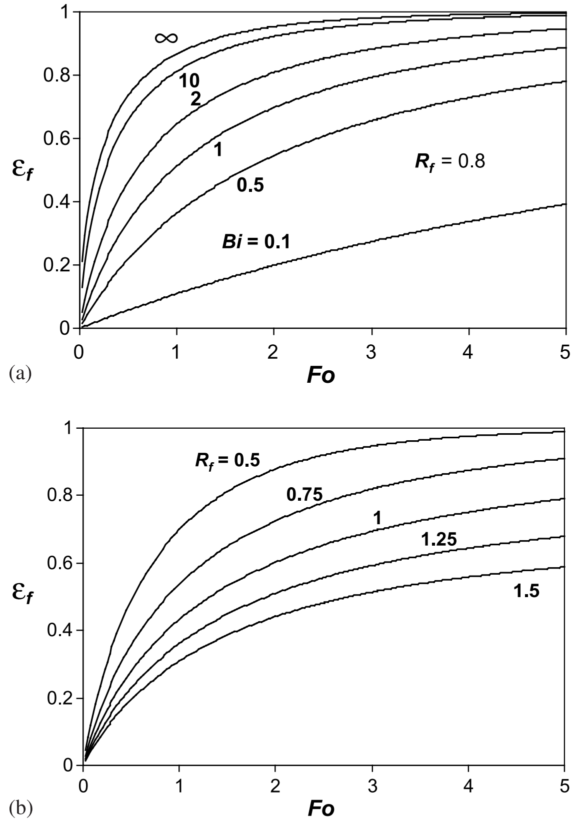

Fig. 5. Effectiveness of heat recovery from a single moving bed ($\Gamma = 0$) to a fluid flow as function of *Fo* with *Bi* (a) and R_f (b) as a parameter ($Bi = 1$).

The approximation, Eq. (44), is useful, since the coefficients μ_1 and K_1 are related to well-known solutions for heating of a solid at constant ambient temperature by convection. They are tabulated for regular shapes (plate, cylinder and a sphere) in many books on heat transfer (see e.g. [26]). Coefficients for some other regular two- or three-dimensional shapes (rod with rectangular cross-section, rectangular parallelepiped, finite cylinder) can be obtained as combinations of the basic case (see e.g. [26,27]). For irregular shapes there are no analytical expression for the coefficients K_1 and μ_1. For these complex shapes the coefficients can be obtained by solving numerically the Fourier equation at constant ambient temperature $\vartheta = 1$ with convection boundary condition. After some time the short-term temperature transients are damped and only the first term ($n = 1$) in the solution $\theta_a = 1 - \sum_n K_n \exp(-\mu_n^2 Fo)$ is required. From numerically calculated values at two times Fo_A and Fo_B, it is possible to solve

$$\mu_1^2 = \ln[(1-\theta_{a,A})/(1-\theta_{a,B})]/(Fo_B - Fo_A) \quad (45)$$

$$K_1 = (1-\theta_{a,A})^{Fo_B/(Fo_A - Fo_A)}/(1-\theta_{a,B})^{Fo_A/(Fo_B - Fo_A)} \quad (46)$$

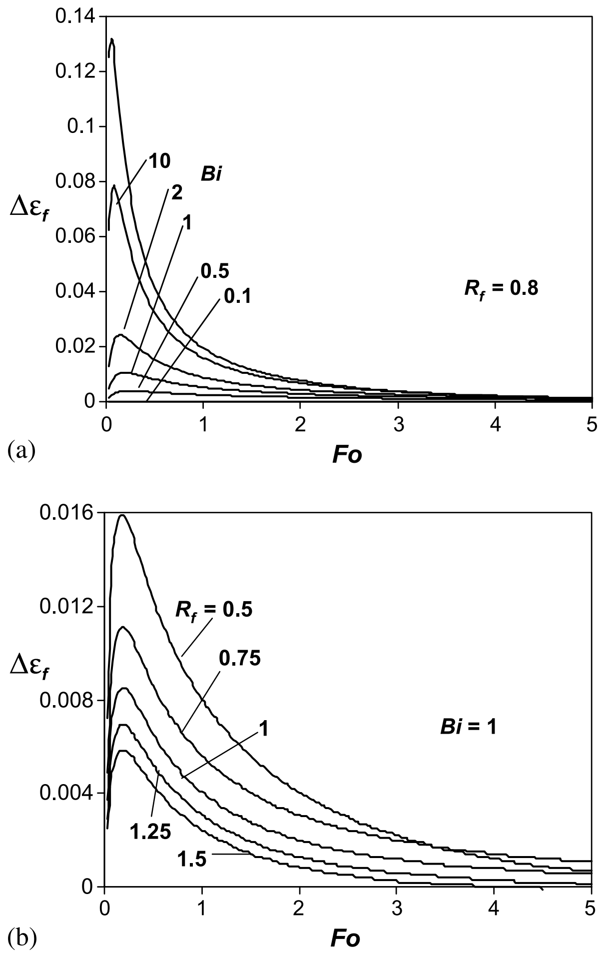

Fig. 6. Comparison of thermal effectiveness for heat recovery from a single moving bed ($\Gamma = 0$) calculated with exact and lumped models as function of *Fo* with *Bi* (a) and R_f (b) as a parameter.

These equations were tested for a plate in the example case $Bi = 5$, for which $\mu_1^2 = 1.726$ and $K_1 = 0.913$. By solving the Fourier equation with finite difference method (21 nodes for a half plate) close values $\mu_1^2 \approx 1.724$ and $K_1 \approx 0.914$ were obtained. Also a two-dimensional case of a rod with a square cross-section was tested (R is half of the side length, $Bi = 5$). For this case the coefficients can be obtained as combination of the values for the plate, $\mu_1^2 = 3.45$ (as the sum) and $K_1 = 0.834$ (as the product). The numerical solution (with a grid 21×21) gave close values $\mu_1^2 \approx 3.44$ and $K_1 \approx 0.836$.

As an example of the irregular two-dimensional element shape, a solid material having ducts with a square cross-section was considered (Fig. 7a), which can be divided in symmetrical elements. The moving bed could be a powdery solid (for example sand) moving in plug-flow manner. The numerically calculated values for μ_1^2 and K_1 are shown in Fig. 7b. These values could also be applied in the simulation of fixed bed heat storage with rectangular channels using Eqs. (14) and (15) for gases (or more exact form for all fluids [12]). After the first transients the temperature distribution, when scaled as $\Theta = (T_p - T_{p,min})/(T_{p,max} - T_{p,min})$, will reach a constant pattern shown in Fig. 7(c and d). This temperature field does not depend on time, since the only term $\exp(-\mu_1^2 Fo)$ affecting will be reduced away due to the definition of Θ. It is also possible to evaluate the coefficient ϕ from the equation $\phi = (\theta_{s,a} - \theta_a)/[2Bi(\vartheta_g - \theta_{s,a})]$ using numerically calculated temperatures. Then ϕ will depend on Bi.

The dimensionless heat transfer areas required for complete phase change for single moving beds with phase change are shown in Fig. 8. It is seen that little less heat transfer area is required for counter-flow compared to parallel flow. In freezing food, this model can be used for example to estimate the required heat transfer area (formed by the surface of the food) to freeze the food

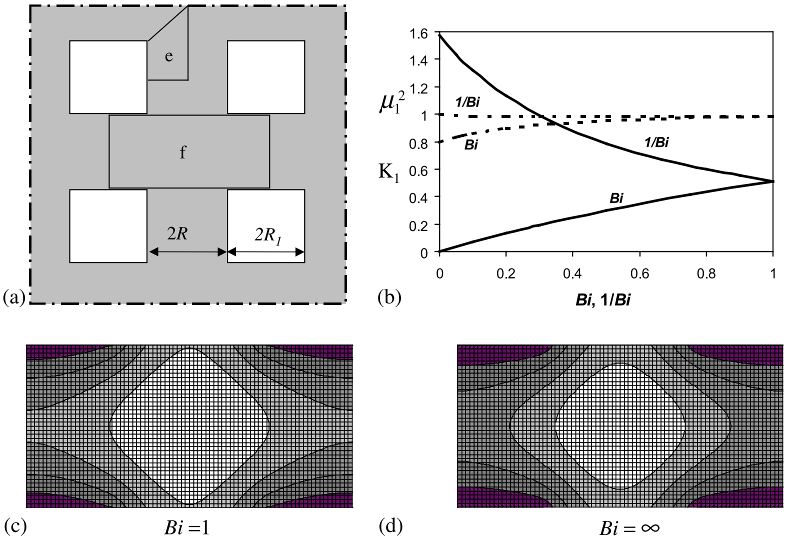

Fig. 7. An example of a complicated element shape. (a) Square passages in a solid medium. Shape of the smallest of the structure is shown by the element denoted by *e*. (b) Numerically calculated values for μ_1^2 with different values of Bi, when $R_1/R = 1$. (c) and (d) The invariant transient long-term temperature distribution Θ ($=$0–0.2, 0.2–0.4, 0.4–0.6, 0.6–0.8, 0.8–1) from the lightest to the darkest in the rectangular region f for two values of Bi.

completely in a tunnel. The heat liberated could be used to melt other food coming from a freeze room, but then a heat pump and possibly a heat storage due to intermittent load is needed.

In some cases due to product quality, an optimum temperature time history or residence time in the moving bed for heating or cooling is required, which determines the size of the bed. It may also be that the products are cooled without simultaneous input flow to be heated. Then, if the heat carrier is liquid, the heat obtained can be stored in a tank for later preheating. If the heat carrier fluid is a gas, a solid sensible or latent heat storage could be applied to balance the time difference of the heat production and need. In a plant with different heat sources the heat storage could be included in the Pinch analysis.

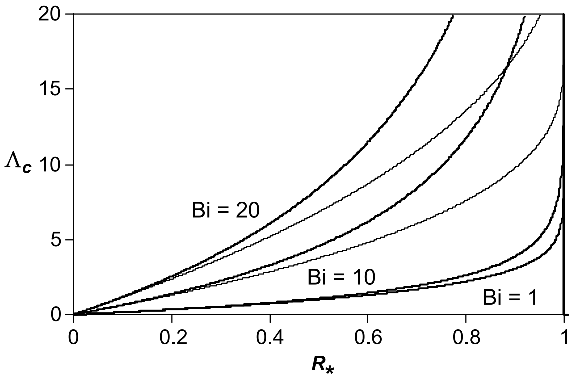

Fig. 8. Dependence of the dimensionless heat transfer area for complete phase change in counter-flow (thin lines) and parallel flow (thick lines) for plates ($\Gamma = 0$).

4.2. Two coupled moving beds

Thermal effectiveness of two moving beds in counter-flow coupled by a fluid flow is shown in Fig. 9 for the symmetric and balanced case ($Bi = Bi_1 = Bi_2, Fo = Fo_1 = Fo_2$). It is seen that the effectiveness reaches maximum close to $R_f = 1$ as predicted by the lumped model. However, calculations show that this relation is not exact, but a very close approximation. Eq. (34) was tested for unbalanced and asymmetric case. In the example case $R_p = 0.6$, min = 1, max = 2, $Fo_1 = 3$, $Bi_1 = 2$, $Fo_2 = 4$ and $Bi_2 = 1$. Then $\Lambda_{min}/\Lambda_{max} = 0.72$ and the optimum value $R_{f1} = 1.303$ is obtained. Eq. (33) with the lumped model, Eq. (19) gives the maximum thermal effectiveness $\varepsilon_1 = 0.767$. Calculations using the exact solution, Eqs. (8), (10) and (33), give almost the same optimum value $R_{f1} = 1.289$ with the maximum thermal effectiveness $\varepsilon_1 = 0.767$.

The indirect heat exchange system, but with two fluids instead of solids, has been considered and economically optimised [15]. The heat transfer surface area is an important economical factor in optimising such system. Now, when heat exchange between fluid and solids (products) is considered, heat transfer area is the surface area of the solid medium itself. In continuous industrial processes this surface area does not usually cause any costs and the main cost factor in the invest-

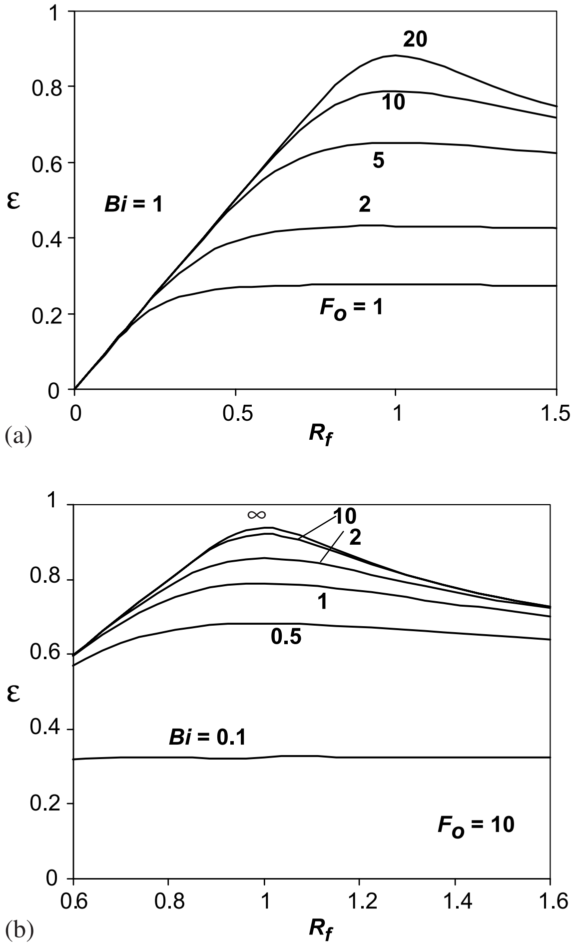

Fig. 9. Thermal effectiveness of two moving beds in counter-flow coupled by a fluid flow with *Fo* (a) and *Bi* (b) as the parameter.

ments is the volume of the vessel containing the moving beds. Thus finding optimum heat transfer for the system consists of two steps: optimising the fluid flow rate and optimising the vessel size. Economic design criteria for cooling of moving beds have been discussed [28].

The thermal effectiveness in the case of cross-flow with fluid mixed between the moving beds using lumped model is shown in Fig. 10 for symmetric and balanced case. The locations of the maximum thermal effectiveness are close to the simple formulas given earlier.

The case with non-mixed flow and thin beds of moving solids (Fig. 4a), the dimensionless conductance between the flows can be presented as

$$\Lambda = (1 - e^{-\Lambda_1} - e^{-\Lambda_2} + e^{-\Lambda_1-\Lambda_2})/(1 - e^{-\Lambda_1-\Lambda_2}) \tag{47}$$

The thermal effectiveness for parallel-, counter- and cross-flow cases are obtained with Eqs. (19) and (22) using this effective value for Λ. The start of the operation of this system is illustrated in Fig. 11 with one bed moving (or both beds in relative movement) can be described by Eq. (21), when ϑ denotes the other solid temperature. Then Eq. (22) can be applied to simulate the start-up.

If the elements consist of tins containing food or bottles containing liquid the lumped analysis is more adequate. The sterilisation of canned products has been considered [29]. A shape coefficient accounting for internal heat conduction could be developed based on such experiments with single canned products. If the bottle contains liquid, then instead of the heat conduction the effect of internal convection should be combined with the surface heat transfer coefficient and the thermal resistance of the wall producing an effective thermal

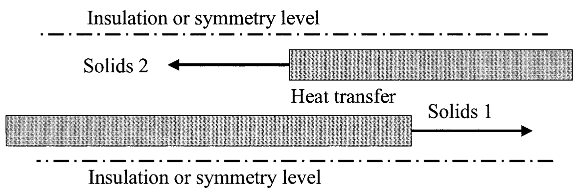

Fig. 11. Heat transfer between two moving beds at start of operation.

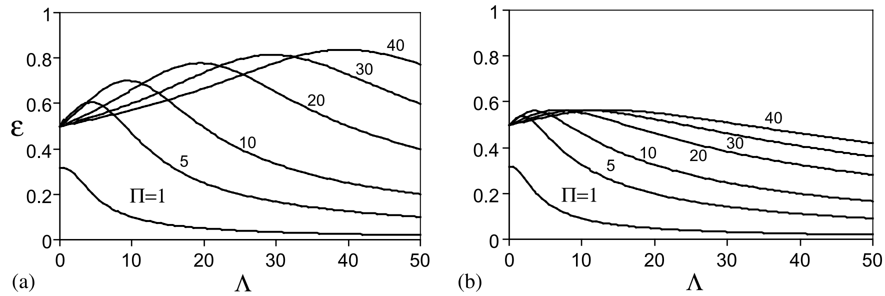

Fig. 10. Effectiveness heat recovery for system of two coupled cross-flow moving beds with fluid completely mixed. (a) Thick bed, Eq. (36). (b) Thin bed, Eq. (38).

conductance h_e. If gas is used as the heat carrier, the thermal resistance is, however, on the gas side and as an approximation the internal resistance can be neglected. The heat transfer in a spherical droplet or vessel due to internal convection has been studied [30,31].

5. Conclusions

In some industrial processes the most natural need for the heat recovered from hot products would be the preheating of the cool solid input. It is possible to use the heat of the hot treated products to preheat cool input. Possible applications are in metal, glass, brick, coke, ceramics and food industries. Analytical solutions for the calculation of heat transfer in single moving beds are presented. The fluid and the solids can be in parallel-, counter- or cross-flow. Solutions accounting for intraparticle heat conduction are compared to an approximate and a lumped model. The approximate model can be applied to complex solid shapes. Heat recovery systems between two solid flows with two coupled moving beds are analysed and optimum conditions to reach maximum thermal effectiveness are discussed.

There is an analogy between two moving beds in cross-flow with fluids unmixed between the beds and the regenerator. In rotary regenerator two fluids are flowing in overall counter-flow and in cross-flow with a moving bed whereas in the heat recovery two moving beds are in counter-flow and in cross-flow with a fluid. Many of the analytical formulas or tabulated results for the thermal effectiveness of regenerators and temperatures can directly be applied to a heat recovery system simply by exchanging the dimensionless space and time co-ordinates and fluid and solid temperatures.

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